Quantum controllers for quantum systems

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Abstract: Feedback control uses sensors to get information about a system, a controller to process that information, and actuators to supply controls. In the conventional picture of quantum feedback and feedforward control, sensors perform measurements on the system, a classical controller processes the results of the measurements, and actuators supply semiclassical potentials to alter the behavior of the quantum system. Since quantum measurement inevitably disturbs the system, the conventional picture portrays quantum feedback control as a stochastic process during which the initial state of the system is destroyed. This paper proposes an alternative method for quantum feedback control, in which the sensors, controller, and actuators are themselves quantum systems and interact coherently with the system to be controlled: as a result, the entire feedback loop is coherent. Feedback control by quantum controllers is not stochastic, preserves the initial state of the controlled system, and can control quantum systems in ways that are not possible using conventional, incoherent feedback control. In particular, the target state to which the quantum controller drives the system can be entangled with another quantum system. This paper investigates quantum controllers and states necessary and sufficient conditions for a Hamiltonian quantum system to be observable and controllable by a quantum controller.

Quantum control theory has a long history (1-4). Experiments on elementary particles, atoms, solid state systems, and optics involve the systematic measurement and manipulation of quantum systems. In addition, control theory has contributed significantly to the understanding of fundamental aspects of quantum mechanics, including the quantum Zeno effect (5-6), non-demolition measurements (7-8), and stochastic quantization (9). Formal aspects of quantum control theory were worked out in the 1980's (1-3, 10), and a host of practical applications of quantum control have been realized in the 1990's as technologies have matured (4,11-14). Particularly significant are experimental applications of optimal control theory to quantum systems (10). Many new technologies in solid state, optical, and atomic physics owe their precision and reliability to quantum control techniques.

The conventional method for controlling a quantum system such as an atom using feedback is to make a measurement on the system to determine its state, then to apply a semiclassical potential such as a laser beam to guide the system to a desired state. This method may be termed semiclassical control. For example, consider an atom in a quantum superposition of its ground and excited state. Stimulated emission can be used to measure whether the atom is in the ground state or in the excited state, leaving it in that state after the measurement. If the atom is found in the excited state, then a laser pulse can be applied to the atom to 'flip' it into its ground state. The net result of this feedback process is to control the atom into its ground state.

From the perspective of control theory, semiclassical control, though effective, has several drawbacks. First of all, measuring a quantum system almost inevitably disturbs it: even an unintrusive measurement that leaves the system in the state in which it was measured — a so-called non-demolition measurement — still alters the state of the system prior to the measurement (7-8). Suppose that the atom of the previous example is originally in a coherent superposition of the ground and excited states. After spontaneous emission determines whether the atom is in its ground state or excited state the initial quantum coherence between those states is irrevocably lost. Secondly, semiclassical control is stochastic: as a result of the measurement the system jumps to one state or another probabilistically.

This paper reports on a new development in feedback and feedforward control of quantum systems. As anyone who has built a PID controller out of op-amps knows, sensors, controllers, and actuators are dynamical systems in their own right. In particular, while the conventional view of quantum feedback control looks at classical controllers interacting with quantum systems through semiclassical sensors and actuators, there is no reason why sensors, controllers, and actuators should not themselves be quantum systems. Quantum sensors and actuators are familiar devices: a photon may get information by

scattering coherently off an electron coherent fashion, and may act coherently on another electron by scattering off it in turn. A controller is a dynamical system that processes the information it gets from sensors and uses that information to make decisions as to what actuators should do. Conventional information-processing devices such as analog or digital computers do not behave in a quantum mechanically coherent fashion. Work on quantum computers (15-18) indicates that both analog and digital quantum devices can process information coherently, however, and simple quantum logic devices have recently been constructed (11-13).

As will be shown below, quantum controllers can perform a number of tasks that semiclassical controllers cannot. For example, they can use coherent feedback to guide a quantum system from an unknown state to a desired state without destroying its initial state. Although the state of the controller becomes correlated with the state of the system during the control process, no irreversible measurement is made, and the initial state of the system is not lost. In addition, a quantum controller can drive a quantum system to a target state that is entangled with another quantum system. Entanglement is a non-local quantum phenomenon that cannot be created by classical controllers.

Consider for example the problem of taking a quantum spin that is originally in the state $|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$, where α and β are unknown, and putting it in the state $|\downarrow\rangle$. In semiclassical control, the controller begins by making a non-demolition measurement of the state of the spin (using, say, a Stern-Gerlach apparatus), giving $|\uparrow\rangle$ with probability $|\alpha|^2$ and $|\downarrow\rangle$ with probability $|\beta|^2$. The control algorithm is as follows: if the result of the measurement is $|\downarrow\rangle$, do nothing, while if the result of the measurement is $|\uparrow\rangle$, put the spin in a static magnetic field B and apply an electromagnetic pulse with frequency $\omega = 2\mu B/\hbar$ to flip the spin (here μ is the spin's magnetic dipole moment). The spin is now in the state $|\downarrow\rangle$ as desired. The control process is stochastic, and although the measurement reveals the state of the spin along some axis, it destroys the original coherent superposition. In the normal idealization of classical feedback and feedforward control, sensors do not disturb the system about which they get information. In semiclassical control of quantum systems, however, measurement introduces an irreversible, stochastic disturbance.

To contrast to the semiclassical case, consider a quantum controller consisting of a second spin, initially in the state $|\downarrow\rangle'$, that interacts with the first through the usual scalar interaction term $\gamma\sigma_z\sigma'_z$ (19-20) so that the Hamiltonian for the two spins is $(\hbar/2)(\omega\sigma_z + \omega'\sigma'_z + \gamma\sigma_z\sigma'_z)$, where $\omega' = 2\mu' B/\hbar \neq \omega$ is the resonant frequency of the second spin. The quantum sensors and actuators operate by enhancing the spin-spin interaction using conventional magnetic resonance techniques. For example, applying a pulse with frequency $\omega' + \gamma$ coherently flips the second spin if and only if spin 1 is in the state $|\uparrow\rangle$. The two spins

are now in the state $\alpha|\uparrow\rangle|\uparrow\rangle' + \beta|\downarrow\rangle|\downarrow\rangle'$. Clearly, the second spin has become correlated with the first spin in the sense that measuring the state of the second spin would reveal the state of the first spin. The state of the first spin has also been disturbed: while initially it was described by the pure-state density matrix

$$\rho = |\psi\rangle\langle\psi| = \alpha\bar{\alpha}|\uparrow\rangle\langle\uparrow| + \alpha\bar{\beta}|\uparrow\rangle\langle\downarrow| + \beta\bar{\alpha}|\downarrow\rangle\langle\uparrow| + \beta\bar{\beta}|\downarrow\rangle\langle\downarrow|,$$

it is now described by the mixed-state density matrix $\rho' = \alpha\bar{\alpha}|\uparrow\rangle\langle\uparrow| + \beta\bar{\beta}|\downarrow\rangle\langle\downarrow|$. No irreversible measurement has taken place, however. The disturbance can be removed and the correlation undone by applying a second pulse with the same frequency to flip the second spin back again, returning both spins to their initial state. With quantum controllers, in contrast with semiclassical controllers, the disturbance introduced by the sensors is reversible and can be undone by the actuators.

A second coherent interaction between the two spins now controls the spin coherently to the state $|\downarrow\rangle$: simply apply to the system in state $\alpha|\uparrow\rangle|\uparrow\rangle' + \beta|\downarrow\rangle|\downarrow\rangle'$ a pulse with frequency $\omega + \gamma$ to flip the first spin if and only if the second spin is up. The state of the two spins is now $|\downarrow\rangle(\alpha|\uparrow\rangle' + \beta|\downarrow\rangle')$. That is, not only has coherent quantum feedback put the first spin in the state $|\downarrow\rangle$, it has coherently put the second spin in the initial state of the first spin. No stochastic operation has taken place, and the initial state of the controlled spin has not been destroyed: rather, it has been coherently transferred to the state of the controller.

Finally, suppose that the goal of the control process is to put the spin in a state $(1/\sqrt{2})(|\uparrow\rangle|\uparrow\rangle'' + |\downarrow\rangle|\downarrow\rangle'')$ where $|\uparrow\rangle''$ and $|\downarrow\rangle''$ are states of a third spin that is not acted on by the controller. Such states are called *entangled* because they exhibit peculiar non-local quantum effects the best known of which is the Einstein-Podolsky-Rosen (EPR) effect (21). Creating and controlling entangled states is a crucial part of new quantum technologies such as quantum cryptography, quantum computation, and teleportation (15-18,22). It is easily verified that the procedure given in the previous two paragraphs accomplishes the goal of producing this entanglement provided that the initial target state stored in the controller is itself entangled with the third spin. The initial state of the three spins is then

$$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)(1/\sqrt{2})(|\uparrow\rangle'|\uparrow\rangle'' + |\downarrow\rangle'|\downarrow\rangle'')$$

and the final state is

$$(1/\sqrt{2})(|\uparrow\rangle|\uparrow\rangle'' + |\downarrow\rangle|\downarrow\rangle'')(\alpha|\uparrow\rangle' + \beta|\downarrow\rangle'),$$

in which the first spin is entangled with the third spin. The controller never acts on the third spin, which may be spatially distant from the first two. A semiclassical controller cannot drive the system to such an entangled target state without acting on the third spin directly.

This example can be generalized to the control of an arbitrary quantum system. Two important questions for any control method (23) are controllability: can a system be guided to a desired state? — and observability: can the sensors determine the state of the system? It is clear from the example above that controllability and observability take on different guises when the controller is semiclassical and when it is quantum-mechanical. We now present necessary and sufficient conditions for controllability and observability of Hamiltonian quantum systems using both semiclassical and quantum controllers.

First, consider semiclassical control of quantum systems. A semiclassical controller applies time-dependent potentials $\sum_i \gamma_i(t)\mathcal{O}_i$ to the system. Controllability is the problem of taking a quantum system from some initial state to a desired final state. A quantum system will be said to be open-loop controllable if the potentials $\gamma_i(t)$ can be chosen to take the system from an arbitrary known initial state $|\psi\rangle$ to a desired final state $|\psi_d\rangle$. This form of controllability is called open-loop because the initial state of the system is assumed to be known, and no measurement is made on the system. The problem of open-loop controllability has an elegant geometric solution (10,17,24,25):

(1) Semiclassical controllability: open-loop case.

A quantum system with Hamiltonian H is open-loop controllable by a semiclassical controller if and only if the algebra $\mathcal{A}$ generated from $\{H, \mathcal{O}_i\}$ by commutation is the full algebra of Hermitian operators for the system.

The spin in the example above is open-loop controllable by a semiclassical controller since NMR methods allow it to be taken from any given state to any desired state: the algebra generated by the Hamiltonian corresponding to the static field, $B\sigma_z$, and the applied Hamiltonian, $B_x\sigma_x \sin\omega t$, can easily be seen to generate the full algebra of $SU(2)$ by commutation. Result (1) is a quantum analog of the geometric theory of classical nonholonomic control theory (26). A familiar example of a classical nonholonomic control problem is parallel parking: a car cannot be driven sideways directly, but can still be parked by edging first in one direction then in another. In the quantum case, the algebra $\mathcal{A}$ determines what set of states can be reached be edging the quantum system first in one direction, then in another, a method that can be called 'parking Schrödinger's car.'

Since the action of the semi-classical controller is Hamiltonian, open-loop semiclassical control is limited to taking a known pure state to a desired pure state. To extend this

controllability result to unknown mixed initial states ρ and to mixed final states ρ_d, we must introduce closed-loop control. Suppose that the controller can make measurements on S (for the sake of simplicity, assume that these measurements are non-demolition measurements) corresponding to a finite set of Hermitian observables $\{\mathcal{M}_j\}$ and then apply potentials $\sum_i \gamma_i(m_j, t)\mathcal{O}_i$ that depend on the results m_j of the measurements. A quantum system S will be said to be closed-loop controllable if and only if a closed-loop controller can take S from an arbitrary unknown initial state ρ to any desired final state ρ_d. We then have the following result:

(2) Semiclassical controllability: closed-loop case.

A quantum system with Hamiltonian H is closed-loop controllable by a semiclassical controller if and only if (i) at least one of the $\mathcal{M}_j \neq I$ — that is, the controller can make some nontrivial measurement on the system— and (ii) the algebra generated by $\{H, \mathcal{O}_i\}$ is the full algebra of Hermitian operators for the system.

For example, the spin above is clearly closed-loop controllable by the semiclassical technique described. The proof of (2) is somewhat detailed (27), and follows from results presented in reference (18). The 'if' part follows because even when one can only make an non-demolition measurement of a single bit of information, the open-loop controllability of the system allows that bit to correspond to projections onto arbitrary subspaces; repeated measurements then allow the value of any operator to be determined and the system to be guided to a desired state. The 'only if' part follows because if the system is not open-loop controllable, then the set of states that can be reached conditioned on the results of measurements is of lower dimension than the Hilbert space of the system.

The close relationship between open- and closed-loop controllability for quantum systems has implications for the related notion of observability. The classical definition of observability must be somewhat altered for quantum systems since the irreversible disturbance introduced by measurement implies that no procedure can reveal the precise initial state of a quantum system. Accordingly, a quantum system will be called observable by a semiclassical controller if the proper sequence of controls and measurements can be used to observe any desired feature of the initial state of the system. Specifically, the system is observable if the controller can make a measurement that reveals the projection of the original state along any desired set of orthogonal axes in Hilbert space. Result (2) immediately implies:

(3) Semiclassical observability.

A Hamiltonian quantum system is observable by a semiclassical controller if and only if it is closed-loop controllable.

In the example above, NMR techniques, together with the ability to measure the component of spin along the z axis, clearly allow one to measure the spin along any axis. In addition, if one can manipulate the spin so as to measure it along any axis, then one can also manipulate it sufficiently to control its state to any desired state, conditioned on the result of the measurement.

Now let's turn to the fully quantum controller. Assume that the system interacts with a quantum controller via a coherent interaction $\sum_i \gamma_i(t)\mathcal{O}_i\mathcal{O}'_i$, where $\mathcal{O}_i, \mathcal{O}'_i$ are Hermitian operators acting on system and controller respectively and the $\gamma_i(t)$ are coupling constants that can be 'turned on' and 'turned off' to make the system and controller interact. We assume that at least one $\mathcal{O}_i\mathcal{O}'_i$ pair is nontrivial in the sense that neither $\mathcal{O}_i$ nor $\mathcal{O}'_i$ is the identity operator: otherwise this case reduces to the semiclassical case above. In analog with ordinary digital control, where the controller is a digital computer with programmable dynamics, the controller is assumed to have access to an arbitrarily large Hilbert space and the dynamics of the controller are assumed to be programmable to any desired dynamics.

For the fully quantum controller, there is no distinction between open- and closed-loop control: an interaction that can function as an actuator can also function as a sensor, and *vice versa*. A quantum system will be said to be controllable by a quantum controller if there is some initial state for the controller (possibly entangled with the state of another quantum system), a dynamics for the controller and a schedule of interactions $\gamma_i(t)$ that takes the system from some initial state ρ to a desired final state ρ_d which can also be entangled with another quantum system. We then have,

(4) Quantum controllability.

A quantum system with Hamiltonian H is controllable by a quantum controller if and only if the algebra $\mathcal{A}$ generated from $\{H, \mathcal{O}_i\}$ by commutation is the full algebra of Hermitian operators for the system.

Result (4) for quantum controllers conflates results (1-2) for semiclassical controllers. Essentially, the equivalence between quantum sensors and quantum actuators makes quantum control intrinisically closed-loop. For example, the one-spin quantum controller above is clearly capable of controlling the other spin to any desired state, entangled or not.

Just as in the semiclassical case, care must be taken in defining observability for quantum controllers: the controller is not a classical device that makes measurements on the system, but a quantum system in its own right that becomes correlated with the

system. No irreversible measurement ever takes place. A quantum system will be said to be observable by a quantum controller if the initial state of the system, together with all its entanglements with any other quantum systems, can be transferred to an analogous state of the controller. The controller can then use this transferred state as the target state to which to control some other quantum system. This fundamentally quantum definition of observability is the natural converse to the quantum definition of controllability in (4). Given results (1-4), the following result should come as no surprise:

(5) Quantum observability.

A Hamiltonian quantum system is observable by a quantum controller if and only if it is controllable by the controller.

As the example of the three spins shows, an interaction with a quantum controller that puts a a Hamiltonian system in a desired state necessarily transfers the initial state or the system, together with its entanglements, to an analogous final state of the controller (28).

In conclusion, this paper explored the properties of semiclassical and quantum controllers and gave necessary and sufficient conditions for quantum controllability and observability, which turned out to be equivalent for Hamiltonian quantum systems. Quantum controllers are likely to play a key role in the development of quantum technologies such as quantum computation and quantum communications. Although the potential experimental realizations of quantum controllers discussed here were based on nuclear magnetic resonance, this paper's results could also be realized using quantum logic devices such as ion traps (12), high-Q cavities in quantum optics (11,13-14), and quantum dots (29). Quantum controllers could have application to a variety of problems, including problems with classical analogs such as trajectory control, and problems with no classical analog such as preventing decoherence. As the theory of quantum error correction shows (30), strategies for disturbance rejection are harder to devise for quantum systems than for classical. A particularly important open question is whether the controllability and observability results reported here for Hamiltonian quantum systems can be extended to open quantum systems.

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